

measurement exercise vi

In addition to being described as beautiful, the experiments of Eratosthenes, Galileo, and Cavendish share another feature: for each, the key discovery involves an argument using proportions. Eratosthenes determined the circumference of the earth using the proportion

$$\frac{\text{angle of shadow at noon}}{360^\circ} = \frac{5000 \text{ stades from Alexandria to Syene}}{\text{circumference of the earth in stades}}$$

Galileo demonstrated that the relative distance traveled by a ball rolling down an inclined plane is equal to the square of the times the take to travel these distances, a proportion that we can write as

$$\frac{\text{distance 2}}{\text{distance 1}} = \left(\frac{\text{time 2}}{\text{time 1}}\right)^2$$

Finally, Cavendish determined the density of the earth by setting up an instrument where the force of attraction between a large weight on a small ball was in relative proportion to the force of attraction between the earth and the small ball.

Eratosthenes' experiment is no more than a logical deduction that yields a simple fact about the earth. Galileo's experiment and Cavendish's experiment, on the other hand, were generalized later into universal mathematical laws by transforming them into equations:

$$\text{distance traveled} = \frac{1}{2} \times a \times \text{time}^2$$

where a is the acceleration due to gravity, and

$$F = G \times \frac{m_1 m_2}{r^2}$$

where F is the force of attraction between two objects, G is the gravitational constant, m_1 and m_2 are the masses of the two objects, and r is the distance separating the objects.

Universal mathematical laws come in two flavors: theoretical laws and empirical laws. A theoretical law is derived from first-principles in which we make some reasoned assumptions and then derive from them a law that we can test experimentally. Galileo's Theorem I, which we reviewed in class, is an example of a theoretical law (or would be if Galileo had written it in mathematical form). An empirical law simply seeks a mathematical equation that explains the relationship between two variables without any claim that it has a basis in theory. An empirical law can, however, show the basic mathematical form that a theoretical law must achieve.

Task 1. What do empirical equations look like?

Task 2. What is a regression analysis?

Task 3. Can we find a model for Galileo's data (as measured by Settle)?