

Integrated Rate Laws and Their Half-Lives

Zero-Order Reaction

rate law

$$R = k$$

differential rate law

$$-\frac{d[A]}{dt} = k$$

derivation of integrated rate law

$$\begin{aligned}d[A] &= -kdt \\ \int d[A] &= -k \int dt \\ [A]_t - [A]_0 &= -kt \\ [A]_t &= [A]_0 - kt\end{aligned}$$

half-life

$$t_{1/2} = \frac{[A]_0}{2k}$$

form of linear plot and how to interpret plot and half-lives

$[A]_t$ vs t ; slope = $-k$; y -intercept = $[A]_0$; each successive half-life is 50% of previous half-life

First-Order Reaction

rate law

$$R = k[A]$$

differential rate law

$$-\frac{d[A]}{dt} = k[A]$$

derivation of integrated rate law

$$\begin{aligned}\frac{1}{[A]}d[A] &= -kdt \\ \int \frac{1}{[A]}d[A] &= -k \int dt \\ \ln[A]_t - \ln[A]_0 &= -kt \\ \ln[A]_t &= \ln[A]_0 - kt\end{aligned}$$

half-life

$$t_{1/2} = \frac{0.693}{k}$$

form of linear plot and how to interpret plot and half-lives

$\ln[A]_t$ vs t ; slope = $-k$; y -intercept = $\ln[A]_0$; each successive half-life equals previous half-life

Second-Order Reaction

rate law

$$R = k[A]^2$$

differential rate law

$$-\frac{d[A]}{dt} = k[A]^2$$

derivation of integrated rate law

$$\begin{aligned}\frac{1}{[A]^2}d[A] &= -kdt \\ \int \frac{1}{[A]^2}d[A] &= -k \int dt \\ -\frac{1}{[A]_t} + \frac{1}{[A]_0} &= -kt \\ \frac{1}{[A]_t} &= \frac{1}{[A]_0} + kt\end{aligned}$$

half-life

$$t_{1/2} = \frac{1}{k[A]_0}$$

form of linear plot and how to interpret plot and half-lives

$\frac{1}{[A]_t}$ vs t ; slope = $+k$; y -intercept = $\frac{1}{[A]_0}$; each successive half-life is $2 \times$ previous half-life